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Laser-Initiated Conical Detonation Wave for Supersonic Combustion. II

F. Fendell,* J. Mitchell,† R. McGregor,‡ and M. Sheffield†
TRW Space and Technology Group, Redondo Beach, California 90278

Further theoretical studies are undertaken of the feasibility of an air-breathing supersonic combustor based on a stabilized, conically configured (oblique) detonation wave. The conical wave is the result of the interaction of a train of spherical detonation waves, each directly initiated by a very rapidly repeated pulsed laser, which is tightly focused on a fixed site in a steady uniform supersonic stream of combustible gaseous mixture. Here, under the idealization of a structureless conical wave, the length of an axisymmetric (nearly conical) nozzle required to exhaust the reacted mixture at ambient-atmosphere pressure is estimated by a steady isentropic ideal-gas flow calculation. For practically interesting flight conditions, a nozzle length of (very roughly) 5 m appears to suffice. Then, the thrust-to-drag ratio achievable with such a combustor, simply enveloped for upper-atmospheric flight, is roughly characterized. However, significant constraints on the range of operation are identified owing to the cellular nature of real detonations. Proof-of-principle laboratory experiments, needed to establish the capacity of existing laser sources to achieve the direct initiation of detonation in hydrogen/air mixtures under conditions of practical interest, and to elucidate further the cellular structure in these mixtures, are outlined.

Nomenclature

$A(x)$ = cross-sectional area of the combustor
a_0 = speed of sound in the unreacted mixture
C_{Dp_c} = inviscid pressure drag coefficient for a cone, Eq. (16)
C_{Dp_s} = inviscid pressure drag coefficient for a sphere, Eq. (18)
C_0 = Chapman-Rubesin factor, Eq. (14)
c_p = specific heat capacity at constant pressure
D = total drag
D_f = portion of drag owing to friction
D_p = portion of drag owing to pressure field
D_2 = second Damkohler number, $q/(c_pT_0)$
E_c = critical energy required for direct initiation of a spherical detonation
F = thrust
L = value of x coordinate at which the conical detonation intersects the combustor wall
L_c = total length of the combustor
L_e = length of the combustor upwind of the energy-deposition site
M = Chapman-Jouguet Mach number for the mixture, u_{CJ}/u_0
M_0 = Mach number of the unreacted mixture, u_0/a_0
p = pressure
q = heat of combustion per mass of mixture
R = gas constant for the mixture, $c_p(\gamma - 1)/\gamma$
Re_{L_c} = Reynolds number based on the length L_c, Eq. (14)
r_n = radius of the partial spherical cap for the combustor sheath
r_{pipe} = (cylindrical) radius of the combustor inlet
s = axial coordinate defined as $x + L_e$
T = temperature
U = axial velocity component (for cylindrical polar coordinates)
u = radial velocity component (for spherical polar coordinates)
u_{CJ} = speed of Chapman-Jouguet detonation
u_0 = axial-velocity component of the unreacted mixture
V_0 = volume of unreacted medium into which energy is deposited
$\mathbf{v}$ = velocity vector
v_∞ = a viscous-interaction parameter, Eq. (14)
w = polar angle velocity component in spherical coordinates
x = axial coordinate, with origin at the site of energy deposition
x' = axial coordinate defined as $x - L$
x'' = axial coordinate defined as $x' - x'_s$
α = $\gamma D_2/(\gamma - 1)$
β = value of θ at which the conical detonation lies
γ = ratio of specific heats of the mixture
θ = spherical polar angle for coordinate system with origin at the energy-deposition site ($\theta = 0$ is directly downwind)
λ = size of a cell in the structure of the detonation
μ = dynamic viscosity
ρ = density
σ = (cylindrical) radial coordinate
τ = shear stress
ψ = r_n/σ_{we}

Subscripts
c = pertaining to a conical enveloping sheath
e = pertaining to the exit plane of the nozzle
s = evaluated at the site at which the conical portion of the nozzle begins
w = evaluated at the combustor wall
0 = pertaining to the unreacted mixture
1 = pertaining to the just-detonated mixture
∞ = pertaining to just-deflagrated mixture

Superscript
$\hat{}$ = unit vector

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*Staff Engineer, Center for Propulsion Technology and Fluid Mechanics. Associate Fellow AIAA.
†Member of the Technical Staff, Center for Propulsion Technology and Fluid Mechanics.
‡Senior Project Engineer, Center for Propulsion Technology and Fluid Mechanics.

I. Introduction

A COMBUSTOR (for high-speed air-breathing propulsion) in which the flow is virtually everywhere supersonic tends to incur less entropy rise, almost inevitably associated with significant flow gradients (e.g., shock waves, shear layers); entropy rise is a measure of the energy unavailable to do useful work. Designs predicated on mixing-controlled, diffusion-flame combustion (the "conventional" scramjet concept[1]) encounter the slow growth of mixing layers in supersonic flow.

Thus, we have considered an alternative supersonic-combustor concept based on the stabilization of a conical detonation wave by means of a train of energy depositing, rapidly repeated laser pulses.[2] Each pulse nonintrusively initiates, at a fixed site relative to the combustor, a spherical detonation in a combustible gaseous mixture flowing faster than its Chapman-Jouguet (CJ) detonation wave speed.[3–6] This concept differs from other oblique detonation wave engine (ODWE) concepts known to the authors[7–15]; other ODWE concepts rely on the introduction of an intrusive body into the supersonically flowing stream of combustible mixture to initiate shock-induced combustion. The introduction of a solid (e.g., conical) body results in the following disadvantages: 1) additional frictional drag occurs along the intruding body (and supporting struts); 2) inevitable body-nose bluntness and body jitter cause a locally normal shock and large entropy rise—perhaps even broadly subsonic flow, since downwind disturbances can propagate upwind in a subsonic flow; and 3) in the absence of a positive ignition device, a long chemical reaction induction time may intercede between the shock induced by the body and the burning of the mixture, especially for the relatively low ambient pressure of higher altitudes—so the combustor may have to be impractically long.

Briefly, we anticipate that, as the fixed finite frequency of detonations is increased from one value to an indefinitely larger value, the entropy increase associated with reflected shocks from the interaction of neighboring detonations becomes progressively smaller.[16] Ultimately, a stabilized conical detonation with negligible "corrugations" arises. This is a standing oblique detonation wave, downwind of which the reacted gas expands in a self-similar way, until the peripheral "edge" of the conical detonation wave interacts with the container wall. If axial position $x = 0$ is the site of energy deposition (Fig. 1) in a circular pipe-type container of radius r_{pipe}, this detonation/wall interaction occurs at downwind distance $x = L \equiv r_{\text{pipe}}/\tan\beta$, $\sin\beta \equiv u_{\text{CJ}}/u_0$, where, by design, the known Chapman-Jouguet detonation wave speed for the mixture $u_{\text{CJ}} < u_0$, the (supersonic) speed of the oncoming mixture. For $x > L$, the flow is no longer self-similar; a (e.g., method-of-characteristics) solution of the isentropic supersonic flow in trial axisymmetric containers of cross section $A(x)$, with $A'(x) > 0$, is undertaken to ascertain whether there exists a nozzle configuration for which, within a relatively short streamwise distance, the detonated mixture may be exhausted at ambient pressure. Calculations are also undertaken to suggest whether a favorable thrust-to-drag ratio is attainable (even after accounting for the decrement from ideal performance owing to real-gas effects).

Thus, the first objective of the work to be reported here is to compute the configuration of a diverging nozzle for exhausting the reacted gas, and to investigate combustor length and thrust-to-drag ratio. These calculations are carried out in Secs. II and III, under the model of a "structureless" conical detonation wave, and, on this "idealized" basis, a high-speed, high-altitude, detonation-wave-based engine might be feasible.

However, when the implications of the observed cellular structure[17–20] of real detonations are adequately accounted for, very severe restrictions on the practical operating conditions for a detonation-wave-based engine arise, and these are identified in Sec. IV.

Finally, in view of the critical constraints imposed by the properties of real detonations, we briefly sketch the type of laboratory experiment required 1) to establish whether existing pulsed-laser sources suffice for the direct initiation of detonation (in gaseous mixtures of practical interest) by nonintrusive energy deposition; and 2) to elucidate further the cellular nature of detonations in these mixtures.

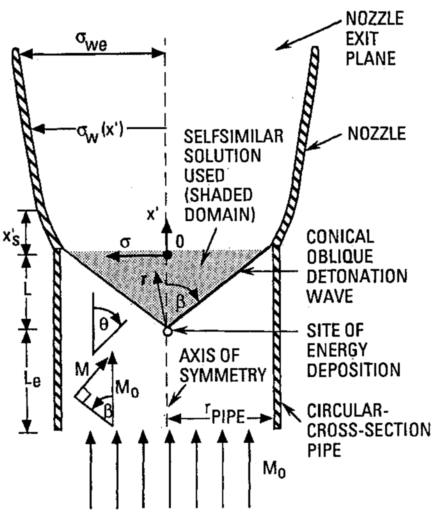

Fig. 1 Schematic (not to scale) of a supersonic combustor, where $\sin\beta = M/M_0$, $M_0 = u_0/a_0$, $M = u_{\text{CJ}}/a_0$, a_0 being the sound speed of the cold mixture and $L = r_{\text{pipe}}/\tan\beta$. The pipe container is flared at the position of intersection with the nearly conical detonation wave, to avoid reflected shocks.

II. Combustor Configuration and Thrust

For a homogeneous combustible mixture of ideal gases encountering a stabilized conical detonation wave, the self-similar postdetonation wave flow[2] is describable entirely in terms of θ, $\beta > \theta > 0$, where β is the half-cone angle of the conical detonation (Fig. 1). The angle θ has its vertex at the energy-deposition site, and $\theta = 0$ is directly downwind. Properties of this hot-gas flow (including, in particular, the angle β) are entirely specified by assignment of values to 1) three dimensionless parameters γ, M_0, and α [these are, respectively, the ratio of specific heats of the mixture (held constant throughout the flow), the Mach number of the cold mixture, and the product of $\gamma/(\gamma - 1)$ times D_2, where D_2 is the ratio of the specific exothermicity of the mixture q to the (specific) static enthalpy of the unreacted mixture c_pT_0]; and 2) two-dimensional parameters p_0 and ρ_0 (these are, respectively, the pressure and density of the cold mixture). Below, in the absence of a study of mixture preparation, we take γ, M_0, p_0, and ρ_0 to characterize the ambient flow as well as the cold combustible mixture.

While the ratio γ is held constant throughout the flowfield in any calculation given below (in fact, for hydrogen-air mixtures in the context of interest, γ varies from 1.4 to about 1.2), we shall examine this range in the numerical calculations to be undertaken. Whereas, γ and M_0 are commonly encountered dimensionless parameters, e.g., in examining a chemically frozen shock, the practically interesting range of the normalized heat of combustion[6] α is perhaps less familiar. If T_∞ denotes the temperature in a gaseous mixture that has undergone combustion and T_0 is the temperature in the unreacted mixture, then $Q \approx c_p(T_\infty - T_0)$, where, for simplicity, the specific heat capacity at constant pressure is approximated as a universal constant. (For a more general development, see Ref. 6.) The equilibrium burned-gas temperature T_∞ for a stoichiometric hydrogen-air mixture at one atmosphere[21] is about 2400 K, and for a stoichiometric hydrogen-oxygen mixture[21] is about 3080 K, for $T_0 = 298$ K. If we adopt, for the ambient speed of sound $a_0 \equiv (\gamma RT_0)^{1/2}$, a value of 350 m/s, and for the heat capacity a lower-bound value of 1 kJ/(kg K), then $\alpha(\equiv \gamma q/a_0^2)$ is equal to about 24 for the stoichiometric hydrogen-air mixture and about 32 for the stoichiometric hydrogen-oxygen mixture. In fact, the heat capacity of a stoichiometric hydrogen-air mixture at room conditions is closer to 1.3 kJ/(kg K), and the heat capacity of superheated steam at 2000 K is about 2.8 kJ/(kg K), so the values of α are slightly larger.

Downwind of the axial site at which the conical detonation wave intersects the circular inlet pipe of radius r_{pipe}, the flow is no longer self-similar, and a cylindrical-polar coordinate system (σ, x') is more convenient than the spherical polar coordinate system (r, θ) (Fig. 2). The axial position of detonation-wave iteraction with the pipe (container) is taken to be $x' = 0$, or $x = L$, since $x \equiv L + x'$, where $\tan \beta = r_{\text{pipe}}/L$. The flow v is thus written alternatively as

$$v = u\hat{r} + w\hat{\theta} = U\hat{x} + v\hat{\sigma} \quad (1)$$

an exception to this convention being that the freestream (axial) speed is written u_0. The site of pulsed-laser energy deposition is $x = 0$, $\sigma = 0$; $0 \leq \sigma \leq \sigma_w(x)$, where $\sigma_W(x)$ is the container wall. Whereas, $\sigma_w(x) = r_{\text{pipe}}$ for $L \geq x \geq 0$, we seek to identify a container configuration $\sigma_w(x')$ for $x' > 0$ (or $x > L$). We seek to discharge, within a "short" streamwise expanse (at $x' = x_e'$, or $x = L + x_e'$), the combusted gas expanded such that the pressure $p(\sigma, x_e') = p_0$, the ambient pressure, for all $\sigma_{we} \geq \sigma \geq 0$, where $\sigma_{we} = \sigma_w(x_e')$. In expanding the burned gas from the known nonuniform state[2] in σ holding at $x' = 0$, to the uniform-pressure state at $x' = x_e'$ where x_e' is to be found), we model the steady axisymmetric supersonic perfect-gas flow as effectively isentropic. We confine our trial-and-error search for an aerodynamically acceptable configuration $\sigma_w(x')$ to one specific family of shapes: $\sigma_w(x')$ follows that known stream-surface (of the postdetonation self-similar flow) that passes through the circle $x' = 0$, $\sigma_w(0) = r_{\text{pipe}}$ to a certain *short* axial distance $x' = x_s'$ (or $x'' = 0$), and $\sigma_w(x')$ is thenceforth composed of the conical diffuser tangent at $x' = x_s'$. Thus, the adoption of a value for x_s' constrains the (one-parameter) family of candidate shapes to be investigated here. We anticipate that $x_e' >> x_s'$; we expect that other families of container configurations permit smaller values of x_e', but our objective is limited to identifying *a* family of containers that permits discharge of the flow at a uniform pressure equal to the ambient pressure. We remark that, as guidance for the iteration, adopting larger values of x_s' helps to "accelerate" the uniformization of the pressure in σ, but the conical-diffuser shape is "more efficient" for decreasing the pressure to ambient levels. As indicated below, all the other principal properties associated with the expansion of the detonated gas to ambient pressure can be readily estimated, so the major motivation for the calculations is to indicate that the nozzle length x_e' is of practical magnitude.

The modified method of characteristics solution of the inviscid equations of motion[2] permits weak pressure waves to propagate from the container wall to the axis of symmetry and back. We shall seek container configurations $\sigma_w(x')$ for various values of γ, M_0, and α, with p_0, T_0, and r_{pipe} fixed. It is quite helpful to track the pressure-wave reflections [typified by an axial expanse in x' in which $p(0, x') < p(\sigma_w, x')$, followed downwind by an axial expanse in which $p(0, x') > p(\sigma_w, x')$, followed downwind by another reversal in the inequality, etc.]. We find that only after a certain invariant number of such reversals (specifically, six) is $p(\sigma, x_e')$ comparably uniform in σ, for various sets of the assigned triplet (γ, M_0, α). A very stringent degree of radial uniformity of pressure (at the ambient value) in identifying x_e' is enforced; a less stringent requirement would have resulted in the acceptance of a still rather uniform (cylindrical-)radial profile of nearly ambient-level pressure achieved at but about half, the axial distance (i.e., at $x' = x_e'/2$).

Further details (concerning the calculation of the initial profiles of the dependent variables in σ for $x' = 0$; of the stream surface through the conical detonation wave/cylindrical pipe intersection; etc.) are available elsewhere.[2] The method of characteristics calculation, with sufficient resolution for current purposes, required about 5–10 min on a Silicon Graphics Iris workstation; no particular effort has been expended to optimize the program.

For the numerical calculations reported throughout this manuscript, we adopt upper-stratospheric values for the ambient thermodynamic state: $p_0 = 100$ Pa, $T_0 = 272$ K. We note that, since (with all other parameters held fixed) all lengths scale with the value that is assigned to the quantity r_{pipe}, the assignment of r_{pipe} is in this sense arbitrary; we adopt the value 0.2 m. The nominal values, defined to be $\gamma = 1.4$, $M_0 = 10$, and $\alpha = 30$ (a value typical of a stoichiometric mixture of hydrogen and air at the values of p_0, T_0 cited), hold in the absence of explicit statement to the contrary.

Figure 3 presents the variation with σ of p and T at the axial positions $x' = 0$ and $x' = x_s'$, which span the short axial expanse (0.211 m) over which the nozzle wall is taken to be coincident with that stream surface of the self-similar flow which passes through $x' = 0$, $\sigma = \sigma_w = r_{\text{pipe}}$. Figure 4 presents corresponding results for U and the radial component v. Figure 5 presents p and T at the axis $\sigma = 0$ and at the wall $\sigma = \sigma_w(x')$ for $x_e' \geq x' \geq x_s'$ [i.e., for $(x_e' - x_s') \geq x'' \geq 0$], as obtained from the method of characteristics numerical integration; Fig. 6 presents corresponding results for U and v, where kinematically $v = 0$ at $\sigma = 0$. Figure 7 presents p and T vs σ at $x'' = 0$ and at $x'' = x_e'' = 4.8$ m, and Fig. 8 presents corresponding results for U and v. Figure 9 presents the var-

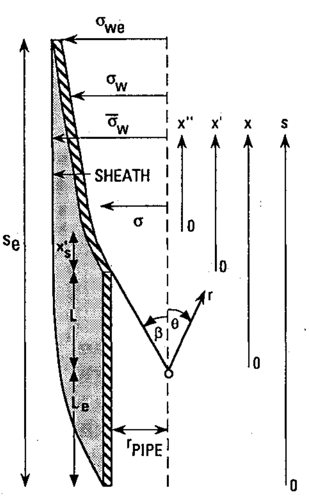

Fig. 2 Coordinate geometry: σ is the cylindrical radial coordinate, with $\sigma = \sigma_w$ at the combustor wall and $\sigma = \bar{\sigma}_w$ at the surface of the enveloping sheath; x, x', x'', and s each denote axial position from a different origin.

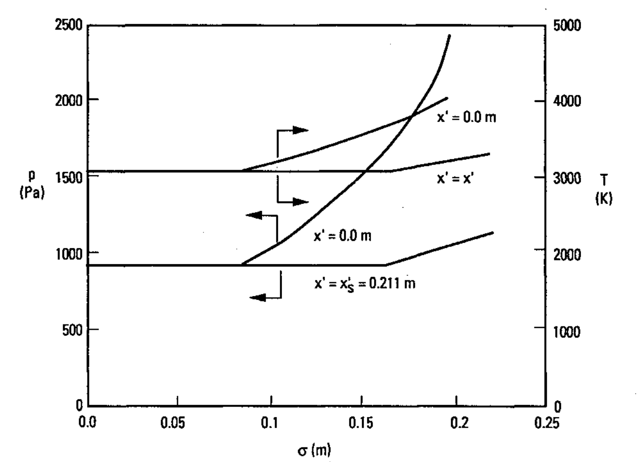

Fig. 3 Nominal conditions ($\alpha = 30$, $M_0 = 10$, $\gamma = 1.4$, $p_0 = 100$ Pa, $T_0 = 272$ K, $r_{\text{pipe}} = 0.2$ m), p and T as function of σ for the two axial positions delimiting the steam surface-wall portion of the nozzle.

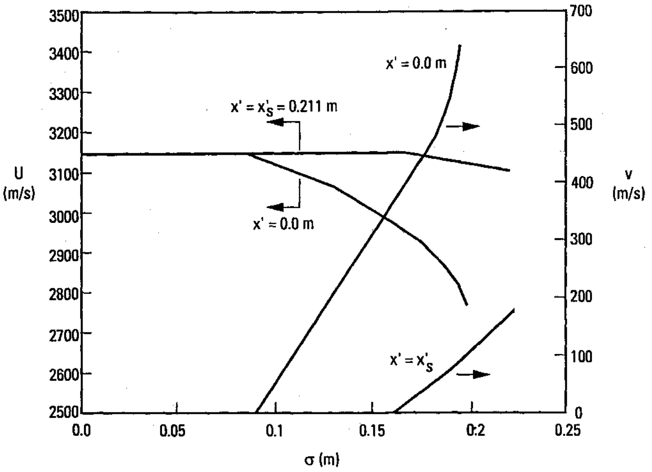

Fig. 4 Same as Fig. 3, but for U and the (cylindrical-)radial component v.

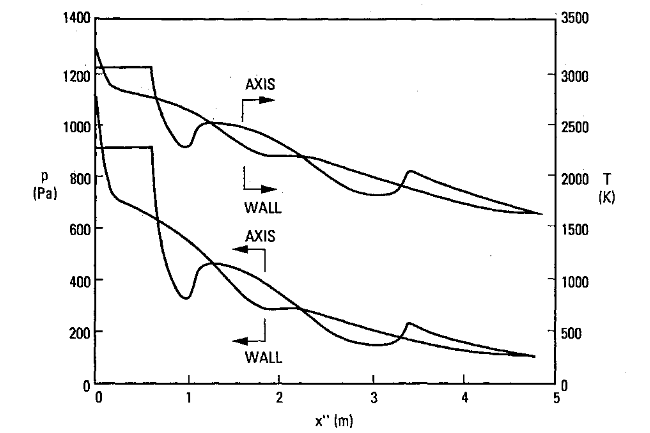

Fig. 5 Over the axial expanse of the conical portion of the nozzle (for nominal conditions), p and T at the axis of symmetry $\sigma = 0$, and at the wall $\sigma = \sigma_w(x')$.

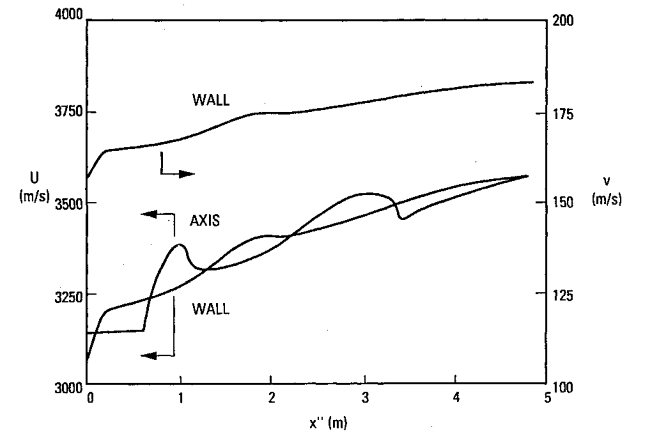

Fig. 6 Same as Fig. 5, but for U and v.

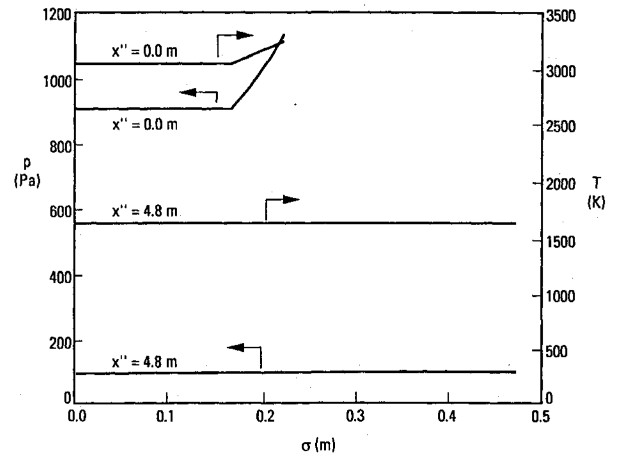

Fig. 7 Nominal conditions, p and T vs σ for the two axial positions delimiting the conical portion of the nozzle.

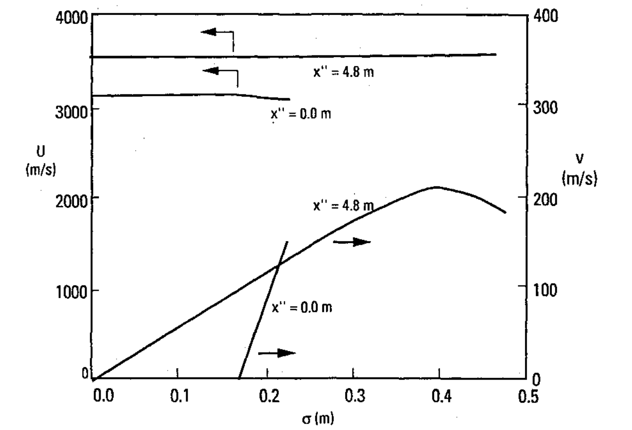

Fig. 8 Same as Fig. 7, but for U and v.

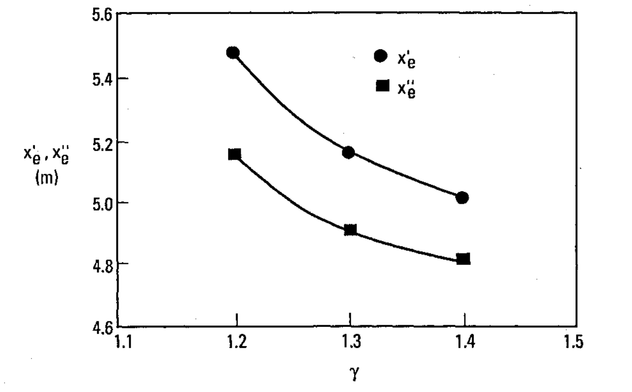

Fig. 9 Axial length of the nozzle x_e', and of the conical portion of the nozzle x_e'', for three values of the isentropic exponent, with other parameters fixed at nominal values.

iation of the (not impractical) axial lengths x_e' and x_e'' [$\equiv(x_e' - x_s')$] for three (fixed) values of the isentropic exponent γ; the corresponding half-angles of the conical portion of the nozzle appear in Fig. 10. Figure 11 presents x_e' and x_e'' for several assignments of the freestream M_0 and the modified second Damkohler number α, and Fig. 12 gives the corresponding half-angles. In summary, we tentatively conclude that the required combustor nozzles, estimated under the above-enumerated simplifications, to be of feasible dimension.

F is given by[22]

$$F = \rho_0 u_0^2(\pi r_{\rm pipe}^2)\left(\bar{\mu}\,\frac{U_e}{u_0} - 1\right) + (\pi\sigma_{we}^2)p_0\left(\frac{p_e}{p_0} - 1\right) \qquad (2)$$

where

$$\bar{\mu} \equiv \frac{\rho_e U_e(\pi\sigma_{we}^2)}{\rho_0 u_0(\pi r_{\rm pipe}^2)} \simeq 1 \qquad (3)$$

since, even for rich mixtures, the increase in mass flux owing to fuel injection is modest for the hydrogen/air case of particular interest here, and, in any case, the details of fuel/air mixing have not been included in the development. Since the pressure at the nozzle exit is equal to the ambient pressure by design [i.e., $p(x_e') \equiv p_e = p_0$], the second term on the right side of Eq. (2) vanishes. Hence, the thrust is identified once the axial velocity component at the nozzle exit, $U(x_e') \equiv U_e$, is computed. At the nozzle exit, the radial velocity component

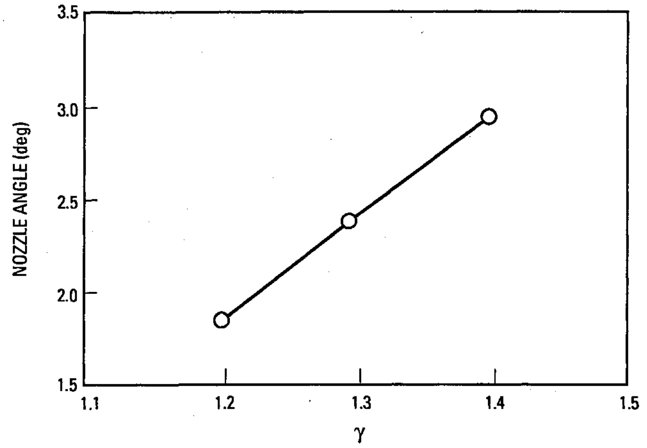

Fig. 10 Half-angles of the conical portion of the nozzle, corresponding to the cases of Fig. 9.

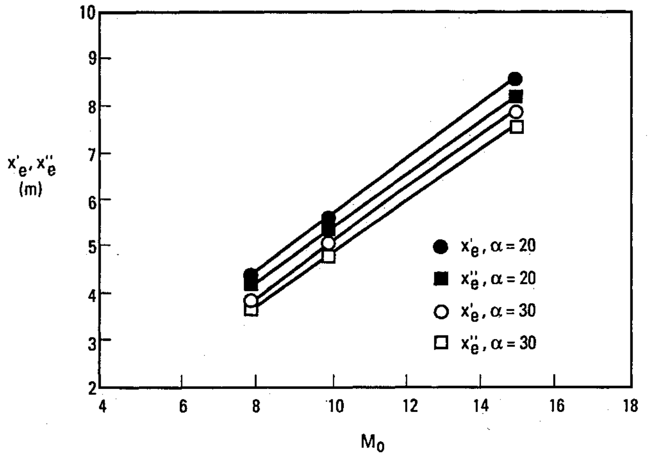

Fig. 11 Same as Fig. 9, but for several values of the cold-mixture M_0 and the modified second Damkohler number α, with other parameters fixed at nominal values.

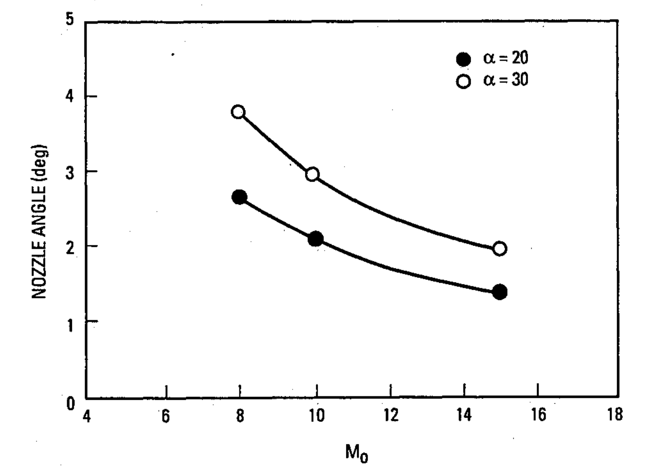

Fig. 12 Half-angles of the conical portion of the nozzle, corresponding to the cases of Fig. 11.

has been ignored relative to the axial velocity component, an excellent approximation for all cases of interest here.

For computing U_e, we denote the conditions[6] holding in the just-denoted gas (i.e., at the conical detonation wave, on its downwind side) by subscript 1

$$\frac{p_1}{p_0} = 1 + \alpha(\gamma - 1)\left\{1 + \left[1 + \frac{2\gamma}{\alpha(\gamma^2 - 1)}\right]^{1/2}\right\} \tag{4}$$

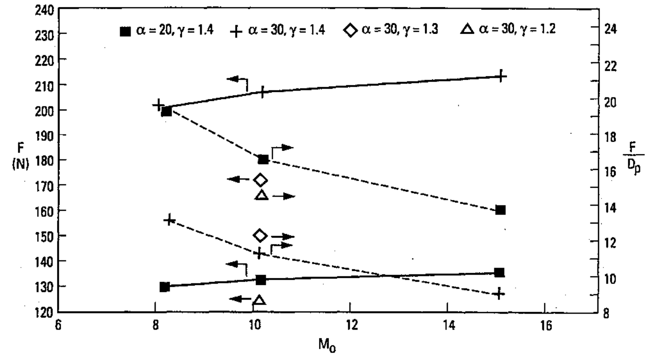

Fig. 13 F and F/D_p for several values of M_0, α, and γ. The combustor sheath is of parabolic configuration [Eq. (11)] and $L_e = 2$ m.

$$\frac{\rho_0}{\rho_1} = 1 + \alpha\,\frac{\gamma - 1}{\gamma}\left\{1 - \left[1 + \frac{2\gamma}{\alpha(\gamma^2 - 1)}\right]^{1/2}\right\} \tag{5}$$

$$M = \left[1 + \frac{\alpha(\gamma^2 - 1)}{2\gamma}\right]^{1/2} + \left[\frac{\alpha(\gamma^2 - 1)}{2\gamma}\right]^{1/2} \tag{6}$$

$$u_1 = (M_0^2 - M^2)^{1/2} a_0, \quad a_0 = (\gamma R T_0)^{1/2} \tag{7}$$

$$w_1^2 = a_1^2 = \gamma R T_1, \quad T_1/T_0 = (p_1/p_0)(\rho_0/\rho_1) \tag{8}$$

Here, we recall that the component of the burned-gas flow speed normal to the CJ detonation, w_1, is at the local sound speed (relative to the detonation). It is also noted (Fig. 1) that the Mach nmber of the component of the cold flow normal to the conical detonation is denoted $M(\equiv u_{CJ}/a_0)$, whereas the Mach number of the cold flow is denoted $M_0(\equiv u_0/a_0)$; the component of the cold flow tangential to the detonation u_1 is continuous across the detonation. For convenience in this very preliminary study, the quantities R, the gas constant for the mixture, and γ are held fixed throughout the flowfield.

The conditions holding at the nozzle exit are taken to be related to the conditions holding just downwind of the detonation by the isentropic and state equations for an ideal gas

$$\rho_e = \rho_1(p_e/p_1)^{1/\gamma}, \quad T_e = T_1(\rho_e/\rho_1)^{\gamma-1}, \quad p_e = \rho_e R T_e \tag{9}$$

By use of Eqs. (4–9), together with the adiabatic relation

$$U_e^2 = 2c_p(T_1 - T_e) + u_1^2 + w_1^2 \tag{10}$$

the quantity U_e in Eq. (2) is now identified in terms of input parameters. Results are presented in Fig. 13. Figure 13 also includes a value for the inviscid pressure drag D_p, for point of reference, although the subject of drag is the topic of the next section. The pressure drag is here computed under the Newtonian approximation [see Eq. (15) below] for an impervious sheath (Fig. 2) enveloping the combustor, $\bar{\sigma}_w(s)$, where the (cylindrical-)radial position of the sheath is taken to be given by

$$\bar{\sigma}_w(s) = r_{\text{pipe}} + (\sigma_{we} - r_{\text{pipe}})\cdot(s/s_e)[2 - (s/s_e)] \tag{11}$$

Such a sheath is coincident with the pipe entry ($s = 0$) and with the nozzle exit ($s = s_e$), and is tangent to the nozzle exit $\{[d\bar{\sigma}_w(s_e)/ds] = 0\}$. For explicitness we take the mixture-preparation length $L_e = 2$ m (Figs. 1 and 2), and it is recalled that $s \equiv x + L_e$. For the above-enumerated nominal conditions, $F \simeq 207$ N and $D_p \simeq 19$ N. The "parabolic" sheath defined by Eq. (11) incurs a notably small value for the pressure drag. In Fig. 13, circumstances for which F is relatively large tend also to be circumstances for which the nozzle-exit radius σ_{we} [and hence, the pressure drag D_p, see Eqs. (11) and (15)] are relatively still larger, so the ratio F/D_p tends to be relatively small.

Incidentally, since, by continuity, the nozzle-exit radius σ_{we}^2 is given by

$$(\sigma_{we}/r_{\text{pipe}})^2 = (\rho_0/\rho_e)(u_0/U_e) \tag{12}$$

we have substantiated the above statement that the only key property of the combustor not readily estimated except by flowfield calculation is the length of the nozzle x_e'.

The calculation of thrust undertaken in this section implies that the ideal enthalpy of reaction is realized. Limitations on the combustion efficiency owing to reaction kinetics (discussed further below in Sec. IV) have been ignored, and this is evidenced by the variation of thrust as a function of M_0 displayed in Fig. 13.

III. Thrust-to-Drag Ratio Considerations

Since we are uncertain what aerodynamic-vehicle configuration would be associated with any application of the supersonic combustor under consideration, we first consider (in relation to the thrust just calculated) the drag associated with alternative, plausibly shaped sheaths enveloping the combustor itself. D on the combustor sheath is taken to be given by the expression

$$D = D_p + D_f \tag{13}$$

Here, to reiterate, D_p is the pressure drag, often estimated for hypersonic flow past simple aerodynamic geometries by the Newtonian approximation; D_f is the friction drag, obtained by use of boundary-layer theory. Adopting this two-part sum as an approximation for D implies that both the viscous-interaction contributions and the transition-to-rarefied flow corrections are relatively unimportant; that is, there is at most a weak viscous interaction (so the adoption of a model in which an effectively inviscid flow occurs between the shock and the boundary layer is suitable over the preponderance of the body), and the molecular mean free path is appreciably smaller than body dimensions. Whitfield and Griffith[23] suggest that such an approximation is viable for slender blunted cones if the viscous-interaction parameter $\bar{v}_\infty \leq 0.1$, where by definition (μ_0 is the ambient-air viscosity)

$$\bar{v}_\infty \equiv M_0\left(\frac{C_0}{Re_{L_c}}\right)^{1/2}, \quad C_0 \equiv \frac{\mu_w}{\mu_0}\frac{T_0}{T_w}, \quad Re_{L_c} \equiv \frac{\rho_0 u_0 L_c}{\mu_0} \tag{14}$$

and where C_0 involves quantities evaluated at the sheath wall (subscript w) and at freestream conditions (subscript 0), and the Reynolds number based on combustor length is denoted Re_{L_c}. For now, we proceed under the simplification that the existence of an axisymmetric pipe-type inlet of finite radius r_{pipe} does not alter the axial-force calculations for an aerodynamically sheathed combustor (of axial length-to-inlet radius ratio of typically 50) from the calculations appropriate for a slender blunted cone. We shall reconsider the matter below. We shall examine sheaths in which a partial spherical cap with a small spherical radius r_n (and with center at $s = r_n$, $\sigma = r_{\text{pipe}}$) is smoothly joined to a cone that then extends precisely to $s = s_e$, $\sigma = \sigma_{we}$ (where it is coincident with nozzle-exit mouth). Here s is an axial coordinate with origin at the inlet to the pipe, and s_e is the axial length to the nozzle-exit plane (Fig. 2): $s_e = x_e' + L + L_e$, where L_e is the axial length of that portion of the inlet pipe (of radius r_{pipe}) that is situated upwind of the laser-energy-deposition site. Hence, s_e is identified with L_c. For convenience, we take the length L_e to scale with the reference length r_{pipe}; probably only experimental testing can establish the suitability of this conjecture. The greater the bluntness of the partial spherical cap, the more suitable is the neglect of viscous interaction that otherwise might arise for small values of s. It is conventional to introduce $\psi \equiv r_n/\sigma_{we}$. We note that McWherter et al.[24] found that neglect of viscous interaction contribution to the zero-lift axial force incurs negligible error for $\bar{v}_\infty \leq 0.0045$, and results in a 15% underestimate for $\bar{v}_\infty = 0.015$; Wilhite et al.[25] make similar comments, and add that transition-to-free-molecular flow effects enter at $\bar{v}_\infty > 0.08$.

The pressure contribution to D_p is approximated by (if $\bar{\sigma}_w(s)$ denotes the cylindrical radius of the sheath)

$$D_p = 2\pi\rho_0 u_0^2 \int_0^{s_e} \frac{\bar{\sigma}_w(s_1)[\mathrm{d}\bar{\sigma}_w(s_1)/\mathrm{d}s_1]^3}{1 + [\mathrm{d}\bar{\sigma}_w(s_1)/\mathrm{d}s_1]^2}\,\mathrm{d}s_1 \tag{15}$$

While direct numerical integration suffices, an approximate result follows.

For a cone of half-vertex angle θ_c with a base of cylindrical radius σ_{we}, if C_{Dp_c} is the inviscid-pressure-drag coefficient for a cone so

$$C_{Dp_c} \equiv 2(p - p_0)/(\rho_0 u_0^2) \tag{16}$$

then

$$D_p = \tfrac{1}{2}\rho_0 u_0(\pi\sigma_{we}^2)C_{Dp_c} = \tfrac{1}{2}\rho_0 u_0^2(\pi\sigma_{we}^2)2\sin^2\theta_0 \tag{17}$$

For a sphere of (spherical) radius r_n, if C_{Dp_s} is the corresponding coefficient for a sphere

$$D_p = \tfrac{1}{2}\rho_0 u_0^2(\pi r_n^2)C_{Dp_s} \doteq \tfrac{1}{2}\rho_0 u_0^2(\pi r_n^2)2[1 - \sin^4\theta_c]$$

$$\doteq \rho_0 u_0^2(\pi r_n^2) \tag{18}$$

since $\sin^4\theta_c \ll 1$ for cases of interest here. The two contributions are, to good approximation, additive, so that for a spherical-cap-blunted slender cone

$$D_p = \rho_0 u_0^2 \pi\sigma_{we}^2(\psi_n^2 + \sin^2\theta_c) \tag{19}$$

$$\sin\theta_c \doteq \sigma_{we}/s_e, \quad \psi_n = r_n/\sigma_{we} \tag{20}$$

The contribution of the nose blunting to the pressure drag is negligible, comparable, or dominant as $(r_n/\sigma_{we})^2$ is negligible, comparable, or large relative to $(\sigma_{we}/s_e)^2$. In view of the presence of the pipe inlet, it might be geometrically appropriate, for purposes of Eqs. (19) and (20), to adjust σ_{we} to take on a value closer to $(\sigma_{we} - r_{\text{pipe}})$. For flight at Mach 10 at $p_0 \simeq 100$ Pa, $T_0 \simeq 272$ K (appropriate to a geopotential height of about 48 km), $\rho_0 \simeq 10^{-3}$ kg/m^3 and $u_0 \simeq 3300$ m/s. If $s_e \simeq 5$ m, and if, as an effective value for purposes of Eqs. (19) and (20) only, $\sigma_{we} \simeq 0.37$ m, then $D_p \simeq 22$ N for $\psi_n = 0$; if instead $r_n \simeq 2.5$ cm so $\psi_n \simeq 0.07$, with all other values held constant, then D_p is approximately doubled. Since the stagnation pressure $\rho_0 u_0^2$ is large, greater bluntness can increase D_p substantially. The pressure drag scales as the square of the reference length (taken to be r_{pipe}), all other (i.e., nongeometric) parameters being held fixed.

D_f is obtained by integrating (over the surface of the sheath) the shear stress at the sheath surface τ_w. From boundary-layer theory for a simple self-similar body (such as a cone), τ_w is given by[23,26]

$$[\tau_w/(\tfrac{1}{2}\rho_w u_e^2)](\rho_w u_e s/\mu_w)^{1/2} = c \tag{21}$$

and $c = 1.15$ for an axisymmetric body. In so writing τ_w, we are ignoring the contribution owing to the nose bluntness and the finite value of r_{pipe}. Thus, the dimensionless frictional-drag coefficient C_{Df} is given by

$$C_{Df} \equiv \left(\frac{D_f}{\tfrac{1}{2}\rho_0 u_0^2 A_{\text{base}}}\right) = \bar{A}\bar{v}_\infty, \quad A_{\text{base}} \equiv \pi\sigma_{we}^2 \tag{22}$$

$$\bar{A} \equiv c\gamma^{1/2}s_e \int_0^1 \left(\frac{p_e}{\rho_0 u_0^2}\right)^{1/2}\left(\frac{u_e}{u_0}\right)^{3/2} \cdot \frac{2\pi\bar{\sigma}_w}{\pi\sigma_{we}^2(s/s_e)^{1/2}}\,\mathrm{d}\left(\frac{s}{s_e}\right) \tag{23}$$

where $\gamma = 1.4$ and subscript e denotes evaluation at the edge of the viscous boundary layer. (The boundary-layer-edge velocity u_e introduced here is not to be confused with the axial velocity component at the nozzle exit U_e, introduced earlier.) Hence, if

$$\bar{\sigma}_w \to s \tan \theta_c, \quad (\sigma_{we}/s_e) = \tan \theta_c \tag{24}$$

then, since the pressure is effectively invariant across the thickness of the sheath boundary layer, and the compressibility factor is about unity

$$\bar{A} = \frac{2c\gamma^{1/2}}{\tan \theta_c} \int_0^1 \left(\frac{p_e}{\rho_0 u_0^2}\right)^{1/2} \left(\frac{u_e}{u_0}\right)^{3/2} \left(\frac{s}{s_e}\right)^{1/2} \mathrm{d}\left(\frac{s}{s_e}\right) \tag{25}$$

In the Newtonian approximation for a cone, $p_e \simeq \rho_0 u_0^2 \sin^2\theta_c \simeq \rho_0 u_0^2 \tan^2\theta_c$

$$\bar{A} \doteq 2c\gamma^{1/2} \int_0^1 \left(\frac{u_e}{u_0}\right)^{3/2} \left(\frac{s}{s_e}\right)^{1/2} \mathrm{d}\left(\frac{s}{s_e}\right) < \frac{4c\gamma^{1/2}}{3} \tag{26}$$

since $u_e < u_0$. The term $(4c\gamma^{1/2}/3) \simeq 1.8$ for $\gamma = 1.4$. If we adopt the upper bound $(u_e/u_0) \to 1$ to compensate for any viscous-interaction effects that we have omitted, then

$$D_f = \tfrac{1}{2}\rho_0 u_0^2 \pi \sigma_{we}^2 \bar{A} v_\infty \to 2.83 \rho_0 u_0^2 \sigma_{we}^2 v_\infty \tag{27}$$

For the nominal case ($\rho_0 \simeq 10^{-3}$ kg/m^3, $u_0 \simeq 3300$ m/s, $\sigma_{we} \simeq 0.5$ m), with $T_w = T_0$ so $C_0 = 1$, $v_\infty \simeq 0.014$ for $s_e \simeq 5$ m, and $D_f \simeq 100$ N. If $(u_e/u_0) = 0.8$, $D_f \simeq 70$ N. Since v_∞ is inversely proportional to the square root of a length, and since obviously σ_{we}^2 is directly proportional to the square of a length, where (for completeness) the reference length is recalled to be r_{pipe}, then D_f scales as the reference length to the three-halves power, all other (i.e., nongeometric) parameters being held fixed.

In summary, we tentatively find that the thrust generated by the supersonic combustor under stratospheric-type conditions is plausibly about twice the sum of the pressure and frictional drag incurred by a slightly blunted, cone configured sheath enveloping the combustor. For such high-speed upper stratospheric flight with modest bluntness of a conical sheath, the dominant contribution to the drag is frictional, and the frictional drag increases as the combustor length to the three-halves power (whereas the thrust is invariant with combustor length). However, we have omitted any contribution to the drag owing to the internal flow within the engine because we have sought no detailed description of the inlet and nozzle flow. This contribution to the drag might negate the net positive thrust.

IV. Operational Constraints Owing to Detonation Structure

Whereas detonations have been idealized as structureless discontinuities in Secs. II and III, observationally, detonations have a cellular structure. Typically, for stoichiometric fuel-air mixtures, the cell size is roughly 10–30 times[6,26] the thickness of the chemical reaction induction zone of the classical Zeldovich-von Neumann-Döring (ZND) structure (a chemically frozen narrow shock, followed by a thicker, virtually nondiffusive deflagration). The cellular nature implies that combustion is completed on a scale related to the flow dynamics. Observationally, no simple relation exists between the induction-length scale and the cellular scale[17,18,26]; hence, we regard theoretical results based on simplistic, one-dimensional, ZND-type structure of uncertain guidance, and prefer to rely on experimental data.

One key observation is that the cell size (which tends to be minimal for an equivalence ratio near unity) varies inversely with the pressure of the undetonated gas, for a mixture of fixed composition.[20] For a stoichiometric mixture of hydrogen and air at atmospheric pressure, the cell size is about 1 cm.[18] Only a stoichiometric mixture of acetylene and air has a smaller cell size. However, whereas hydrogen has many advantages as a fuel for high-speed vehicles, acetylene can be unstable and seems impractical. A second key observation is that the detonative mode of combustion is not stable within an enclosure unless the geometric scale of the enclosure accommodates several cells.[18] The several practical implications of these observations are now considered.

First, if the pressure of the unreacted hydrogen-air mixture were one-tenth of ambient atmospheric pressure, the cell size of the detonation would be about 10 cm. If we take the pressure to vary directly with the density in an isothermal atmosphere—for a very rough characterization of the stratification of the atmosphere, then the freestream M_0, γ, and the exothermicity group α remain fairly invariant with change of atmospheric pressure, and the nozzle length of roughly 5 m continues to hold for a 20-cm-radius inlet pipe. The number of cells compatible with stable burning might not be accommodated by the adopted pipe radius for 10-kPa pressure in the mixture about to be detonated, and, in any case, the adequacy of treating the detonation front as a discontinuity becomes problematic. If we were to adopt a 100-cm-radius inlet pipe to accommodate the enhanced cell size, then the nozzle length would be 25 m, and therefore, probably implausible in both length and exit radius. Thus, vehicle operation such that the mixture about to be detonated is closer to one atmosphere warrants consideration.

Second, the minimal energy E_c required for the direct initiation of a spherical detonation is conservatively taken[16] to be approximately equal to $(\rho_0 q V_0)$. Here, ρ_0 is the ambient density, q is the exothermicity per mass of mixture, and V_0, the volume into which the initiation energy is (nonintrusively) deposited, is taken to be $4\pi\lambda^3$. This seems roughly the plausible scale required for deposition [well within a time scale (λ/u_{CJ})]. (Lee[18] has derived a requirement that the critical energy be equal to about 125 times this amount, but his model is predicated on the decay of an overdriven blast, and may set excessive requirements for more efficient modes of coupling initiation energy into the medium to be detonated.) In any case, the product $(\rho_0 q V_0)$ is equal to roughly 30 J for a stoichiometric hydrogen-air mixture at atmospheric pressure. This requirement is stringent because, among existing lasers, only exceptional CO_2 lasers and perhaps a few others can produce 30 J per pulse, and no existing laser pulsed can produce this energy per pulse with a rapid repetition rate. However, the requirement still may be excessive because it may not be necessary to irradiate a volume based on the entire cell size λ. Experiments (considered briefly in the next section) may reveal that it may suffice for direct initiation to bring a volume, based on a spatial scale smaller than the cell size, up to the temperature achievable by releasing the chemical energy of that portion of the medium. In any case, for the previously discussed isothermal atmosphere in which pressure varies directly with density, since we recall that $\lambda \sim p_0^{-1}$, the critical energy $E_c \sim p_0^{-2}$ if the cellular scale is at all relevant for initiation. [In fact, even if the (by common definition,[27] nearly isothermal) reaction-induction scale of the hypothetical ZND detonation were relevant, for an overall binary reaction (reasonable for hydrocarbon-air combustion), this scale also varies inversely as the pressure,[21,27] so again,[27] $E_c \sim p_0^{-2}$.] Thus, an already demanding energy-per-pulse requirement for nonintrusive direct initiation of detonation becomes significantly more demanding with a decrease in pressure from nearly one atmosphere in the mixture about to be detonated.

Third, since the cell size becomes long for highly subatmospheric pressure in the mixture about to be detonated, both the energy yield and the derived thrust may depart from the values computed on the basis of equilibrium chemistry. The nozzle-exit velocity may be reduced because the finite rate of the chemical kinetics can result in freezing of a nonequilibrium gas composition. Any reduction in combustion efficiency is particularly likely to be associated with the relatively slow three-body recombination of reaction intermediate species at subatmospheric pressure.[28] Incomplete reaction as the flow undergoes expansion in the nozzle is another problem possibly encountered because of the cell-size increase if operation is attempted at highly subatmospheric pressure.

If the mixture about to be detonated is approximately at one atmosphere, then the ambient air should not be at highly subatmospheric pressures for the following reasons. We seek to minimize compressional preheating associated with the mixing of the fuel and air upwind of the laser energy deposition site, lest premature ignition occur. We anticipate that such premature ignition probably precludes the feasibility of the detonation wave engine concept for very high Mach number, very high attitude conditions. Even if the preheating does not incur premature ignition, there is some experimental evidence that the stoichiometric range of detonable mixtures for a practically interesting fuel/oxidizer pair may be contracted.[20] Also, theoretical results suggest that, for fixed pressure in a stoichiometric mixture, the induction length for a detonation increases slightly with increased temperature of the unreacted mixture.[27] More importantly, even if the preheating does not incur premature ignition, the detonated mixture may be brought to such a temperature that the combustion products are chemically frozen in a highly dissociated state when rapidly expanded for exhaust to the ambient; if so, the derived exothermicity is significantly reduced.[28]

Thus, detonation cell size considerations, the requirement that the speed of the about to be detonated mixture speed exceed the Chapman-Jouguet speed, and the trend that high-speed flight at low altitude incurs excessive frictional drag and heating, all suggest that the conical detonation wave engine seems restricted to the lower hypersonic, not-too-high-altitude flight domain.

V. Proof-of-Principle Experiments

Purely computational simulation of the intricate physical phenomena involved in the direct nonintrusive initiation of detonations, and the interaction of such detonations with one another and with solid-container surfaces, seem to us to have very limited credibility. Rather, we suggest that, in a spherical bomb with optical access for very high-speed photography and with very fast-response pressure gauge instrumentation, the following results be sought:

1) The identification of the nonintrusive energy deposition, the volume of deposition, the time interval of deposition, and (if necessary) the sensitization of the mixture (via trace-species additives) to the incident laser-pulse radiation, sufficient for the direct initiation of a spherical detonation in various fuel/air mixtures of practically interesting stoichiometry, composition, and (possibly subatmospheric) initial pressure.

2) The observation of the interaction of two spherical detonations, one initiated just after (but in very close spatial proximity to) the other, so that the detonations are of unequal radius.

3) The observation of the interaction of a spherical detonation wave with a solid container wall, with emphasis on noise, vibration, and general structural response of the wall.

The first task establishes the optical source requirement for the direct nonintrusive initiation of detonation[29]—a key enabling technology. It also permits us to inquire into the cellular nature of spherical detonations and to clarify certain physical concepts (e.g., is the minimum critical volume of deposition of the previously mentioned cellular scale, or smaller; is the minimum deposition energy comparable to the chemical exothermicity derivable from the irradiated volume within the detonable volume; and is the critical time interval for energy deposition effectively equal to the time for a pressure wave to propagate across the irradiated volume).

The second task elucidates whether, in the envisioned supersonic combustor, any unanticipated entropy-producing phenomena arise from the interaction of successive spherical detonations (beyond the entropy increase associated with the reflected shocks that are an anticipated aspect of the interaction).

The third task inquires whether any such unanticipated entropy producing phenomena (beyond the reflected shock) arise from detonation/wall interaction. The last task seems of the lowest priority because we anticipate that we can shape the container so that we avoid such structural problems; nevertheless, the matter warrants clarification.

Such experiments seem to be the highest priority next step for establishing in principle the feasibility of a supersonic combustor based on a laser-initiated conical detonation wave. Experiments in which a conical wave is stabilized in a stream of detonable mixture flowing faster than its Chapman-Jouguet wave speed are clearly more demanding. Such futuristic experiments require the existence of a rapidly repeated pulsed laser, or synchronized lasers focused on a common spot, such that a frequency probably near 1 MHz is achieved. Each pulse must succeed in the direct initiated of a spherical detonation. Until the optical source requirements are ascertained, conjectures on the portability of an appropriate source seem premature.

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